

# GENERALIZED MICROECONOMICS

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# **Generalized Microeconomics**

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## FOREWORD

This publication summarizes the results of more than ten years of theoretical research in the field of microeconomics at the Faculty of Social Sciences at Charles University in Prague. The objective of this research was to generalize microeconomics so as to enable modelling of economic rationality even in fields that standard microeconomics more or less avoids. These fields are not insignificant. For example, roughly half the financial transactions in a modern economy (including donor activities) involve redistribution. The profit maximization assumption makes it impossible to gain a microeconomic modelling insight into centrally planned economies and above all into the non-profit sectors of market economies. The same can be said for externalities (both positive and negative).

In our view, abandoning the *homo economicus* paradigm—in the sense of replacing it with a different paradigm with a different (alternative) agent criterion function that conflicts with profit maximization—is an impassable route and one that bypasses the treasure trove of knowledge of standard economics.

We have opted for a different path: we try to broaden the scope of microeconomics in order to capture the activity of non-profit institutions while treating standard profit/utility maximization as a special case. In other words, instead of abandoning the *homo economicus* paradigm, we generalize it. This generalization complements rather than challenges standard microeconomics. Where the *homo economicus* modelling approach can reasonably be applied, we do not feel the need to abandon it. We venture beyond the boundaries of this standard microeconomic paradigm primarily where non-profit institutions operate and where, simultaneously, economic activity can be both rational and irrational.

For us, the generalizing criterion is “Darwinian” maximization of the probability of survival. This criterion is not necessarily considered explicitly by individual agents in their everyday decision-making, but if they do not respect it they will not survive in the long run.



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## THE GENERALIZED PRINCIPLE OF ECONOMIC RATIONALITY

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The decision-optimization principle contained in the *homo economicus* paradigm conceals an assumed preference for a situation lying on the very boundary of the set of feasible solutions. Unless a *homo economicus* agent (a model producer or a model consumer) can estimate how the parameters of his decision-making problem are going to evolve, he will opt for a situation lying on the boundary of his production or consumption possibilities.

This is perhaps one of the most contentious aspects of neoclassical microeconomics, since producers, for example, will in reality tend to have a legitimate distrust of, or even aversion to, extreme situations located at the limits of technological or financial feasibility and will therefore prefer production situations that lie inside the set of feasible solutions. Being at the boundary is risky, as even a small change in the parameters of a decision-making problem can generate technological inconsistency.

An even stronger preference for a solution that is an internal point of the set of feasible solutions can be assumed in the case of legal constraints. Balancing on the boundary of legal admissibility usually entails a lot of extra non-productive effort and costs. This applies most of all to small firms, which cannot afford expensive lawyers.

The decision-taker also has to ensure that his behaviour is understood by others and does not disrespect established practices. Here again, maximization of profit (personal gain) in accordance with the *homo economicus* model behaviour leads inevitably to situations lying on the boundary of social and moral

admissibility, situations where cooperation collapses, social relations become chaotic, and conflicts and disputes break out with such frequency that resolving them can hardly be described as efficient expenditure of human energy and other scarce resources.

In our view, the standard *homo economicus* economic paradigm does not offer enough scope to cover all the ways in which economic agents behave. In line with Sen, we cannot accept the economic behaviour described by the *homo economicus* paradigm as a requirement for rationality of economic agents.<sup>1</sup>

Efforts to cover a wider context than the purely liberal neoclassical paradigm are not new, of course. In the next section we mention (briefly and without aiming to be comprehensive) some of the trends in economic theory in this sense.

## 1.1 ALTERNATIVES TO THE HOMO ECONOMICUS PARADIGM

We have already discussed the standard decision-making principle used in neoclassical microeconomics, according to which an agent chooses—rationally and perfectly—the option with the highest subjective utility from the set of feasible decisions available and is capable of implementing that decision.

One alternative to this standard decision-making principle is the satisfaction principle, also known as the bounded rationality principle,<sup>2</sup> which assumes that agents do not seek the optimal option forever: the search process is terminated as soon as a satisfactory solution has been found.

Another alternative to the standard decision-making principle is the concept of cognitive dissonance in an individual's rationality. This assumes that agents' rationality fails and that some agents systematically introduce errors, mistakes and distortions into their decision-making processes when considering past experience.<sup>3</sup> Cognitive bounding of rationality therefore essentially represents the consequences of human flaws (such as procrastination).

Another alternative to the standard decision-making principle is the concept of "hard-core" altruism, where an agent incorporates the utility of other agents, or other members of society, into his decision-making motives.<sup>4</sup>

There is also a series of model modifications of the neoclassical paradigm within the framework of the standard decision-making principle. Perhaps the best known is the labour-managed firm (LMF) for cooperatives, in which the

1 Sen, A.: *On Ethics and Economics*. Oxford: Blackwell, 1987, p. 16.

2 Simon, H. A.: Theories of Bounded Rationality. In *Decision and Organisation*, edited by C. B. McGuire, R. Radner, 161–76. Amsterdam: North Holland, 1972.

3 Akerlof, G. A.: Procrastination and Obedience. *American Economic Review* 81, 2(1991): 1–19.

4 A review of these concepts can be found in Hlaváček, J. et al.: *Mikroekonomie souděžitosti se společenstvím*. Praha: Karolinum, 1999.

same group of people plays the role of both owners and employees. This model assumes that an LMF maximizes income per capita, where income is the sum of wages and personal income stemming from profit.<sup>5</sup>

Another way to extend the calculation of profit within the standard decision-making principle is to take into account the extent and magnitude of the effort exerted by managers.<sup>6</sup>

A further approach that does not involve abandoning the standard decision-making principle is the superintendent criterion constructed by Benjamin Ward in an attempt to describe the socialist planned economy.<sup>7</sup> The same can be said for the “*homo se assecurans*” model, where the producer’s maximization criterion is the margin between its ability to produce and the output it actually produces. Chapter 6 of this book will be devoted to this model. The “employee escape” model represents another attempt to model and describe a centrally planned economy with typical excess demand in the labour market.<sup>8</sup>

The application of game theory, which takes into account the active existence of other economic agents and the predictable effects of their decisions on the firm’s decisions, can also be regarded as an example of generalization within the standard decision-making principle. The same goes for models describing agents’ efforts to acquire positional goods, or social status.<sup>9</sup> Buchanan’s concept of club goods is also a generalization of the standard economic paradigm.<sup>10</sup>

Even the concept we present in this book, in which we try to construct a general model of economic behaviour, does not abandon the standard decision-making principle. As in mainstream economic theory, we assume that a decision-taker (economic agent) prefers (explicitly or implicitly through his decision) the economic action that he considers to be the best from his perspective, and has information on the consequences of all the possible feasible decisions.

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5 See Vanek, J.: *The General Theory of Labor-Managed Market Economies*. Ithaca: Cornell University Press, 1970.

6 See Hunter, H.: Optimal Tautness in Development Planning. *Economic Development and Cultural Change* 9, 4(1961), 561–72, or Keren, M.: On the Tautness of Plans. *Review of Economic Studies* 39, 4(1972): 469–86.

7 See Ward, B.: *The Socialist Economy*. New York: Random House, or Hlaváček, J., Tříška, D.: *Úvod do mikroekonomické analýzy*. Praha: Fakulta sociálních věd UK, 1991, pp. 101–7.

8 Hlaváček, J., Zieleniec, J.: Trh práce v ekonomice, přecházející od plánu k trhu—teoretická východiska. VP No. 379. Praha: Ekonomický ústav ČSAV, 1991, pp. 21–23.

9 Becker, G. S.: The Theory of Social Interactions. *Journal of Political Economy* 82, 6(1974): 817–26.

10 For more details see section 10.2.1.4.

## 1.2 MINIMIZATION OF THE SUBJECTIVE PROBABILITY OF ECONOMIC EXTINCTION

If we admit that the economic criterion arises as a result of Darwinian natural selection, every successful economic agent (i.e. every agent that survives in the long run) tries (at least intuitively) to avoid situations involving a high risk of extinction. Therefore, we have chosen minimization of the (subjective) probability of extinction as the agent's general decision-making criterion. It can be assumed that in a liberal market environment such a criterion will be established by natural selection: agents that do not behave in this way will become extinct.

If a decision-taker feels that a low amount of funds is the sole threat to his existence, he will react to this threat with economic behaviour that can be explained using the standard neoclassical *homo economicus* paradigm, i.e. he will maximize his profit or disposable income.

If the individual feels that inferior social status is part of the threat, he will endeavour to increase his social prestige (i.e. to augment his human and social capital, in Becker's terminology). A non-profit university threatened by loss of accreditation because professors are leaving their posts will reduce this risk by increasing their pay. An individual who feels that a threat to other members of society is a threat to society as a whole and therefore also to himself will eliminate this perceived threat by behaving altruistically in society.

An economic agent usually faces not just one threat, but numerous different ones. If a producer's profit is too low, its owner may depart or it may go bankrupt. If its wages are too low, its employees may quit or the quality of its workforce may fall too low. If its price is too high, its sales may be too low. If its share of the market is too small, it may not be able to sign a sales agreement with a monopsonistic buyer. Its managers may instinctively reject a rapid change in production conditions as an inestimable risk. From the manager's point of view, operating at the upper limits of the firm's production capacity (on the production function) may be risky: if the parameters of the firm's economic situation (which the manager cannot fully control) change only slightly, he will not be able to meet the owners' expectations and he may risk losing his lucrative position in the firm and his reputation as a successful manager (for example for failing to deliver the expected profit).

The various threats perceived by a decision-taker or a group (managers, employees, owners) involved in settings the economic agent's criterion are often simultaneous and sometimes contradictory. If an agent knows how to estimate his probability of economic extinction for each individual threat, he can combine those probabilities (for example by summing them if the threats are mutually independent), thereby converting all the threats into a single scalar cardinal criterion, namely the probability of extinction of the agent due to materialization of any of the threats under consideration. Such a criterion, com-



binning all the threats perceived by the decision-taker, then often leads to the optimal solution within the set of feasible solutions of the model. This optimal solution is often a trade-off.

Suppose that an agent's survival (or the threat to his existence) depends solely on his income, or rather on his income relative to the subsistence level: the closer the agent is to the subsistence level, the higher is his probability of (economic) extinction and so the stronger is his subjective feeling of being personally threatened.

Like profit (but unlike consumer utility), the subjective probability of personal survival is a cardinal utility function. In deterministic models we can get by with an ordinal utility function. However, in situations of a stochastic nature (such as the St. Petersburg paradox covered in Chapter 2 or the principal-agent problem discussed in Chapter 3) we cannot get by with an ordinal utility function and we can view a cardinal criterion as being an advantage in this regard.

In most chapters we will assume that the subjective probability of survival is directly proportional to the margin relative to the boundary of the extinction zone (i.e. relative to the subsistence level). This assumption is consistent with an asymmetric Pareto probability distribution.

### 1.3 PARETO DISTRIBUTION OF THE PROBABILITY OF SURVIVAL

The Pareto probability distribution was originally intended to represent the allocation of wealth in an economy. Later on it was used to describe, among other things, the health structure of populations of individuals, the uneven distribution of human settlement, the frequency of occurrence of individual words in a text when decoding secret messages, and the size distribution of sources or deposits of raw materials. In physics it has been used to describe certain phenomena at temperatures close to absolute zero. In all these applications it has the advantage of being asymmetric.

#### 1.3.1 FIRST-ORDER PARETO PROBABILITY DISTRIBUTION

If we assume that an agent's probability of survival is directly proportional to the ratio of his margin (relative to the extinction zone boundary  $b$ ) to his income  $d$ , we arrive at a first-order Pareto probability distribution<sup>11</sup> with the asymmetric distribution function:

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11 Outside economics the first-order Pareto probability distribution is sometimes called the Bradford distribution.

$$F(d)=0 \quad \text{for } d \leq b,$$

$$F(d)=\frac{d-b}{d} \quad \text{for } d > b.$$

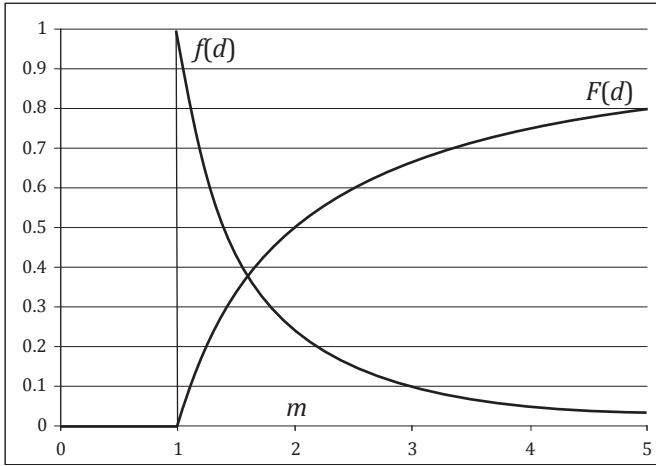
The probability density function for this probability distribution has the following shape:

$$f(d)=0 \quad \text{for } d < b,$$

$$f(d)=\frac{b}{d^2} \quad \text{for } d \geq b.$$

The plots of the probability distribution function  $F(d)$  and the probability density function  $f(d)$  for the first-order Pareto probability distribution with a unit extinction zone boundary  $b$  are shown in Figure 1.

**Figure 1: The first-order Pareto probability distribution with certain-extinction-zone boundary  $b = 1$**



The first-order Pareto probability distribution has a zero probability for income at or below the subsistence level  $b$  and a probability converging to one as income tends to infinity. Unlike higher-order Pareto distributions, the first-order Pareto distribution does not have a final mean or variance. Its median is  $m = 2b$ .

We use the first-order Pareto distribution to express the subjective probability of survival in most chapters of our book. Only in the final chapter, where preferences are the deciding factor for the survival of politicians and those preferences are linked to growth in (rather than the level of) the standard of

living, do we work with the assumption that the probability of survival is directly proportional to the derivative of the relative margin with respect to income. This assumption is consistent with the second-order Pareto probability distribution.

### 1.3.2 SECOND-ORDER PARETO PROBABILITY DISTRIBUTION

According to the psychological Weber–Fechner law<sup>12</sup> individuals in many cases decide not according to the intensity of a stimulus, but according to the change in the intensity of the stimulus. Individuals' assessment of their own satisfaction is often derived from the dynamics rather than the level of a utility indicator (wealth, threat): people in societies with low but rising living standards paradoxically tend to be more satisfied than those in societies with higher but flat or falling living standards. The incorporation of this law into the problem of economic threat (or the subjective feeling of threat) leads to the assumption that the subjective estimate of the probability of personal extinction is linked

not directly with the relative margin  $1 - \frac{b}{d}$ , but with its derivative  $\left(1 - \frac{b}{d}\right)' = \frac{b}{d^2}$ .

So, if it is true that the determining factor for the strength of the subjective feeling of threat is the increase (decrease) in the margin relative to the subsistence level in response to a (small) unit change in income, the second-order Pareto probability distribution is the right one to use for the distribution of the subjective probability of extinction. For this distribution it holds that the risk of extinction decreases in proportion to the square of the distance from the extinction zone.<sup>13</sup> In this case the distribution function representing the probability of survival is

$$F(d) = 0 \quad \text{for } d < b,$$

$$F(d) = 1 - \left(\frac{b}{d}\right)^2 \quad \text{for } d \geq b.$$

and the probability density function for this distribution has the following shape:

$$f(d) = 0 \quad \text{for } d < b,$$

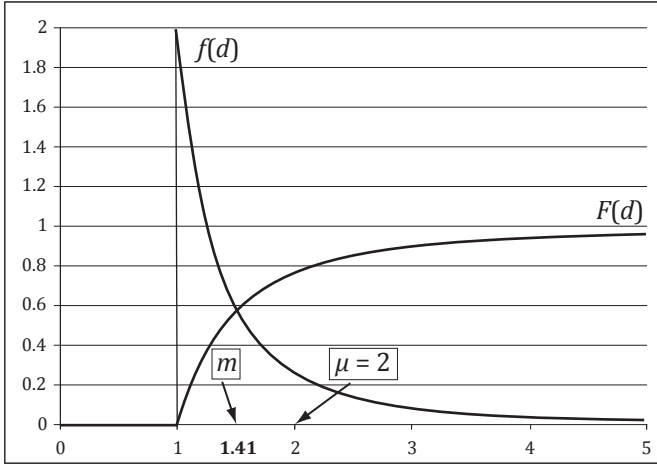
12 See, for example, Frank, R. H.: *Microeconomics and Behavior*. New York: McGraw-Hill, 1994, chapter 8, p. 276.

13 Whereas for the first-order Pareto distribution the risk of extinction decreases in proportion to the distance from the extinction zone.

$$f(d) = \frac{2}{b} \cdot \left(\frac{b}{d}\right)^3 \quad \text{for } d \geq b.$$

Figure 2 shows the probability density function  $f(d)$  and the distribution function  $F(d)$  for the second-order Pareto distribution.

**Figure 2: The probability density function  $f(d)$  and distribution function  $F(d)$  for a second-order Pareto distribution with extinction-zone boundary  $b = 1$**



The second-order Pareto distribution has a zero probability for income not exceeding the boundary of the survival zone and a probability converging to one as income tends to infinity. It has mean  $\mu = 2b$  and median  $m = b \cdot \sqrt{2}$ . This distribution does not have a final variance.

### 1.3.3 GENERAL PARETO PROBABILITY DISTRIBUTION

The general Pareto distribution of order  $\alpha^{14}$  with boundary  $b$  has the distribution function

$$F(d) = 0 \quad \text{for } d < b,$$

$$F(d) = 1 - \left(\frac{b}{d}\right)^\alpha \quad \text{for } d \geq b.$$

14 When used for the distribution of wealth this parameter is called the Pareto index.

The probability density function of this distribution has the shape:

$$f(d) = \frac{\alpha}{b} \cdot \left(\frac{b}{d}\right)^{\alpha+1} \quad \text{for } d \geq b,$$

$$f(d) = 0 \quad \text{for } d < b.$$

The mean for second- and higher-order Pareto distributions is

$$\mu = \frac{\alpha \cdot b}{\alpha - 1}.$$

The standard deviation of a Pareto distribution of order  $\alpha \geq 3$  is

$$\sigma = \left(\frac{b}{\alpha - 1}\right)^2 \cdot \frac{\alpha}{\alpha - 2}.$$

We obtain the Dirac delta function  $\delta(d - b)$  from the  $\alpha$ -th-order Pareto distribution function as the limiting case for  $\alpha \rightarrow \infty$ .

The following figure compares Pareto distributions of various orders and the Dirac delta function:

**Figure 3: Comparison of the characteristics of Pareto distributions of orders 1, 2 and 3 with extinction zone boundary  $b = 1$  (the dotted line shows the Dirac delta function  $\delta(d - b)$ )**

